조합 응력상태의 해석

Metal Forming CAE Lab.
Department of Mechanical Engineering
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선형계의 구조해석

⊙ 선형계

- ○ 선형계: 작은 변형, 후크법칙을 따르는 재료
- ○ 중첩의 원리: 각 하중에 대한 결과(변위, 응력)를 각각 구하여 최종적으로 더해 주면 됨

⊙ 구조설계

- ○ 구조: 자동차, 선박, 항공기, 일반기계, 전자장치, 건물, 부품 등 목적을 가진 모든 고체
- ○ 설계의 목적: 기능 만족, 품질 고급화, 비용의 최소화, 상황에 타라 무게 최소화 등등
- ○ 설계 시 고려사항: 변위 및 변형률, 응력, 고유진동수, 수명 등등

⊙ 구조해석

구조설계, CAD

재료의 성질 $(E, \nu, \alpha, \rho, \cdots)$
하중 및 경계조건

→ **구조해석** →

정적특성: 변위, 변형률, 응력
동적특성: 고유진동수, 고유모드

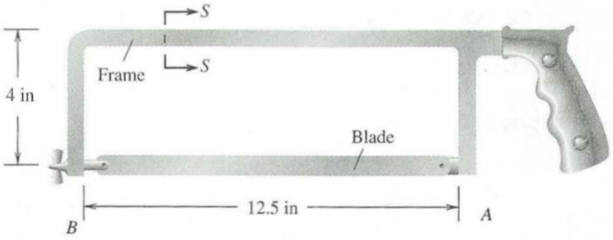
수계산: 역학현상의 이해에 필수

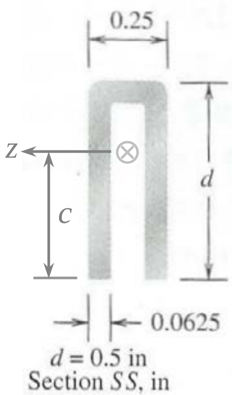
수치해석: 유한요소법, CAE

축력과 굽힘모멘트 조합하중

⊙ 예제 1

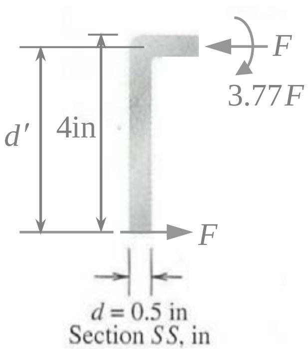
○ 주어진 값과 조건





$$A = 7.03\times10^{-2}\ \text{in}^2$$
$$I_{zz} = 1.64\times10^{-3}\ \text{in}^4$$
$$c = 0.274\,\text{in}$$
$$d = 0.5\,\text{in}$$
$$d' = 3.77\,\text{in}$$
$$\sigma_a = 30\,\text{ksi}$$

○ 단면에 작용하는 하중의 계산



- 축력 : $-F$
- 굽힘모멘트 : $-3.77F$

○ 응력의 계산

- Axial stress due to F

$$\sigma_{xa} = -\frac{F}{A} = -14.2F$$

- Bending stress

$$\sigma_{xb_{\min}} = -\frac{(-3.77F)\times(-0.274)}{I_{zz}} = -630F$$

- Combined stress

$$\sigma_{x_{\min}} = \sigma_{xa} + \sigma_{xb_{\min}} = -14.2F - 630F = -644F$$

○ 하중 F 의 결정

$$\sigma_{x_{\min}} = -Y \;\rightarrow\; -644F = -30\times10^3\ \text{lb}/\text{in}^2$$

$$\therefore\ F = 46.6\ \text{lb}$$

○ 설계의 변경[$d = 0.5'' \rightarrow 0.6''$]의 영향

$A = 8.28\times10^{-2}\ \text{in}^2$, $I_z = 2.76\times10^{-3}\ \text{in}^4$

$c = 0.325\,\text{in}$, $d' = 3.725\,\text{in}$

마찬가지 방법으로,

$$\sigma_{x_{\min}} = -12.1F - 439F = -30\times10^3 \rightarrow F = 66.5\ \text{lb}$$

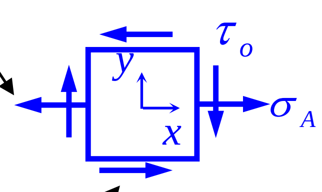
비틀림모멘트와 축력의 조합하중

⊙ 예제 2

○ 축력에 의한 응력

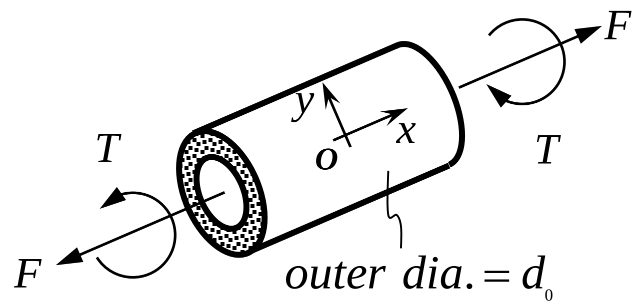
$$\sigma_A = \frac{F}{A}$$

$$\sigma_A = \sigma_x$$



$$\tau_{xy} = -\tau_o$$

$$\tau_o = \frac{T \cdot d_o}{2J}$$

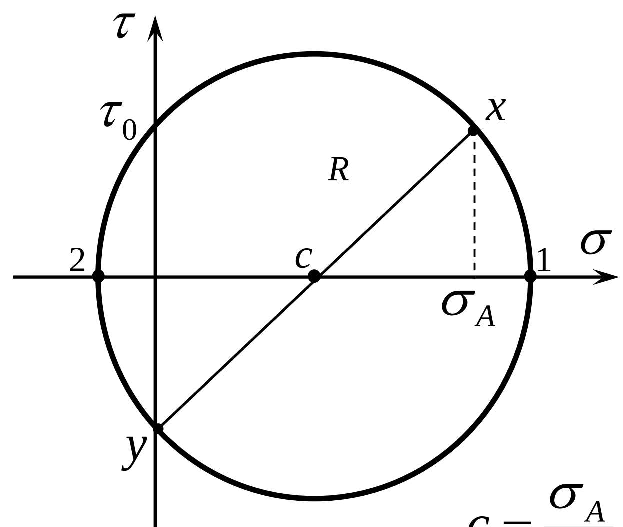


$$outer\ dia. = d_0$$

$$inner\ dia. = d_i$$

$$J = \frac{\pi}{32}\left(d_0^{\ 4} - d_i^{\ 4}\right)$$

$$A = \frac{\pi}{4}\left(d_0^{\ 2} - d_i^{\ 2}\right)$$



$$c = \frac{\sigma_A}{2}$$

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_o^{\ 2}}$$

$$\sigma_1 = c + R,\ \sigma_2 = c - R$$

○ 비틀림모멘트에 의한 응력

비틀림모멘트와 축력의 조합하중

⊙ 예제 3

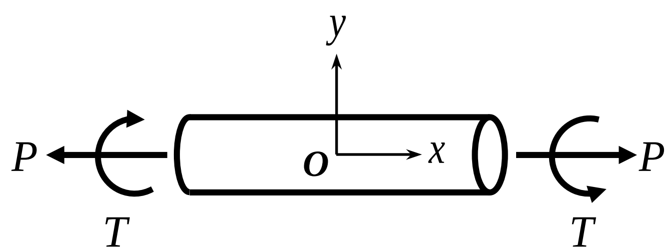
○ 주어진 값

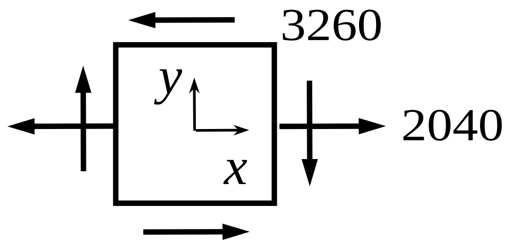
$$d = \frac{5}{16}''$$

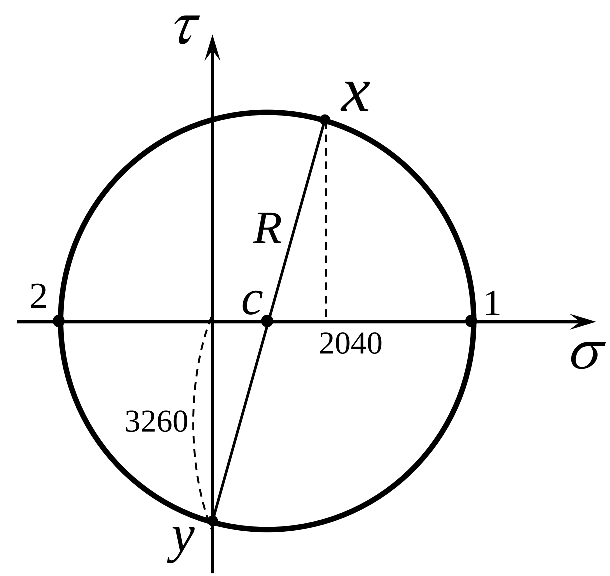
$$T = 10\ \text{lb}\cdot\text{in}$$

$$P = 100\ \text{lb}$$

○ 문제의 정의 및 좌표계 설정







○ 응력의 계산

$$\sigma_x = \frac{P}{\pi d^2 / 4} \approx 2040\ \text{psi}$$

$$\tau_{xy} = -\frac{Td/2}{\pi d^4 / 32} = -\frac{16T}{\pi d^3} \approx -3260\ \text{psi}$$

$$c = 1020, \quad R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^{\ 2}} \approx 3420$$

○ 주응력과 최대전단응력의 계산

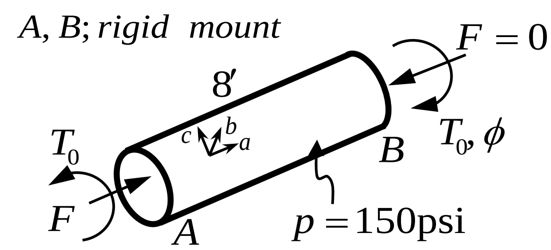
$$\sigma_1 = 2400\ \text{psi} \qquad \leftarrow \quad c + R$$

$$\sigma_2 = -4440\ \text{psi} \qquad \leftarrow \quad c - R$$

$$\tau_{max} = R = 3420\ \text{psi}$$

비틀림모멘트와 축력의 조합하중

⊙ 예제 4



○ 주어진 값 또는 조건

$$\begin{pmatrix} \varepsilon_a = 48\times10^{-6}\,, \quad \varepsilon_b = 343\times10^{-6}\,, \quad \varepsilon_c = 204\times10^{-6} \\ E = 30\times10^{6}\,\text{psi}\,, \quad v = 0.3\,, \quad d_0 = 12''\,, \quad t = 0.125'' \end{pmatrix}$$

○ 측정된 변형률로부터 응력의 계산

$$\begin{pmatrix} \varepsilon_{x'}(0) = \varepsilon_x = \varepsilon_a \\ \varepsilon_{x'}(45^\circ) = \dfrac{\varepsilon_x+\varepsilon_y}{2} + \dfrac{\gamma_{xy}}{2} = \varepsilon_b \\ \varepsilon_{x'}(90^\circ) = \varepsilon_y = \varepsilon_c \end{pmatrix}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

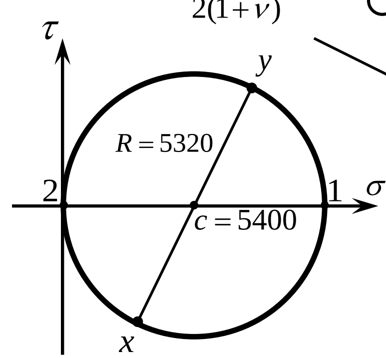
$$\begin{pmatrix} \varepsilon_x = 48\times10^{-6},\ \varepsilon_y = 204\times10^{-6} \\ \gamma_{xy} = 434\times10^{-6} \end{pmatrix}$$

$$\begin{pmatrix} \sigma_x = \dfrac{E}{1-v^2}\left(\varepsilon_x + v\varepsilon_y\right) = 3600\,\text{psi} \\ \sigma_y = \dfrac{E}{1-v^2}\left(\varepsilon_y + v\varepsilon_x\right) = 7200\,\text{psi} \\ \tau_{xy} = \dfrac{E}{2(1+v)}\gamma_{xy} = 5010\,\text{psi} \end{pmatrix}$$

$$\tau_{xy} = \frac{T_0 d_0}{2J}$$

$$J = \frac{\pi}{32}\left(d_0^{\ 4} - d_i^{\ 4}\right)$$

$$G = \frac{E}{2(1+\nu)}$$



○ 내압에 의한 응력의 계산

$$\sigma_H = \frac{rp}{t} = 7200 = \sigma_y,\ \sigma_A = \frac{\sigma_H}{2} = 3600 = \sigma_x$$

○ 전단응력을 이용한 비틀림모멘트의 결정

$$T_0 = \frac{2\tau_{xy}J}{d_0} = \frac{2\times5010}{12}\times\frac{\pi}{32}\times\left(12^4 - 11.75^4\right)$$

$$= 137\,\text{kip}\cdot\text{in} = 137\times10^3\,\text{lb}\cdot\text{in}$$

○ 기타

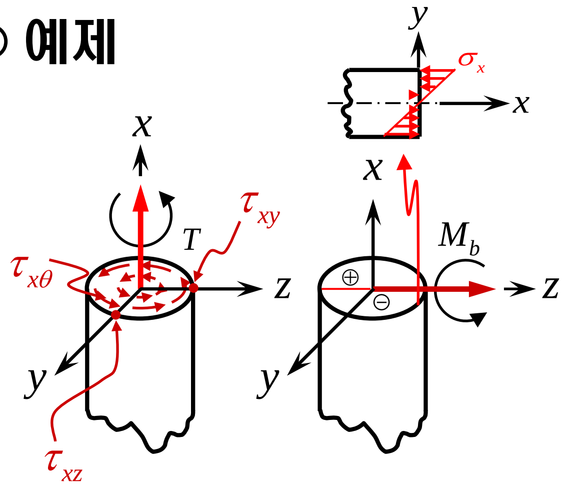
$$\phi = \frac{T_0 L}{GJ} = \frac{\left(137\times10^3\right)\times8\times12}{\left(11.5\times10^6\right)\times164.4} = 6.96\times10^{-3}$$

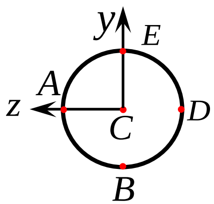
$$\sigma_1 = 10720\,\text{psi},\ \sigma_2 = 80\,\text{psi},\ \sigma_3 = 0$$

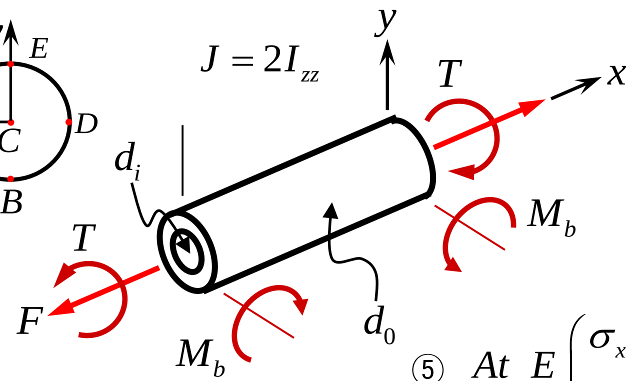
$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 5360\,\text{psi}$$

굽힘모멘트와 비틀림모멘트의 복합하중

⊙ 예제



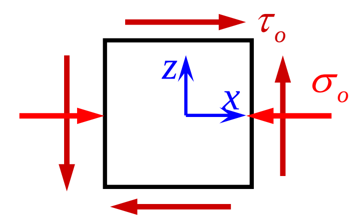




$$\tau_{\theta z} = \frac{Tr}{2I_{zz}}, \quad \sigma_{xy} = -\frac{M_b y}{I_{zz}}$$

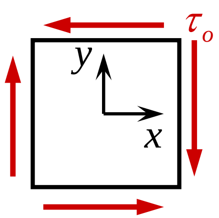
$$I_{zz} = \frac{\pi}{60}\left(d_0^{\,4} - d_i^{\,4}\right)$$

⑤ At E $\begin{pmatrix} \sigma_x = -\sigma_0 \\ \tau_{xz} = \tau_0 \end{pmatrix}$



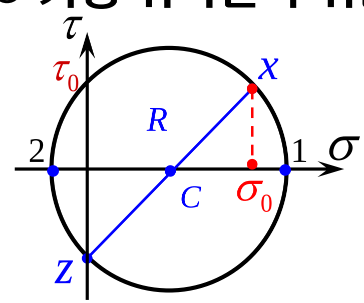
○ 가장 취약한 부위는 점 B 와 점 E 임

① At A $\begin{pmatrix} \sigma_x = 0 \\ \tau_{xy} = -\tau_0 \end{pmatrix}$



$$\sigma_0 = \frac{M_b r_0}{I_{zz}}$$

$$\tau_0 = \frac{Tr_0}{2I_{zz}}$$

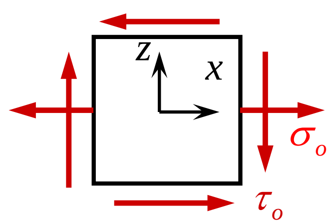


$$R = \sqrt{\sigma_0^{\,2}/4 + \tau_0^{\,2}}$$

$$\sigma_1 = \frac{\sigma_0}{2} + R, \quad \sigma_2 = \frac{\sigma_0}{2} - R$$

$$\sigma_3 = 0$$

② At B $\begin{pmatrix} \sigma_x = \sigma_0 \\ \tau_{xz} = -\tau_0 \end{pmatrix}$



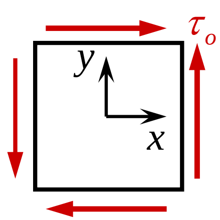
○ 항복이론의 적용

$$\tau_{max} = R = \frac{Y}{2} \rightarrow \sqrt{\left(\frac{M_b r_0}{2I_{zz}}\right)^2 + \left(\frac{Tr_0}{2I_{zz}}\right)^2} = \frac{Y}{2}$$ ← Tresca

③ At C $\begin{pmatrix} \sigma_x = 0 \\ \tau_{xy} = 0 \end{pmatrix}$ ×

$$\bar{\sigma} = Y \rightarrow \sqrt{\frac{1}{2}\left[4R^2 + R^2 + \frac{\sigma_0^{\,2}}{4} + R^2 + \frac{\sigma_0^{\,2}}{4}\right]} = \sqrt{\sigma_0^{\,2} + 3\tau_0^{\,2}} = Y$$ ← von Mises

④ At D $\begin{pmatrix} \sigma_x = 0 \\ \tau_{xy} = \tau_0 \end{pmatrix}$

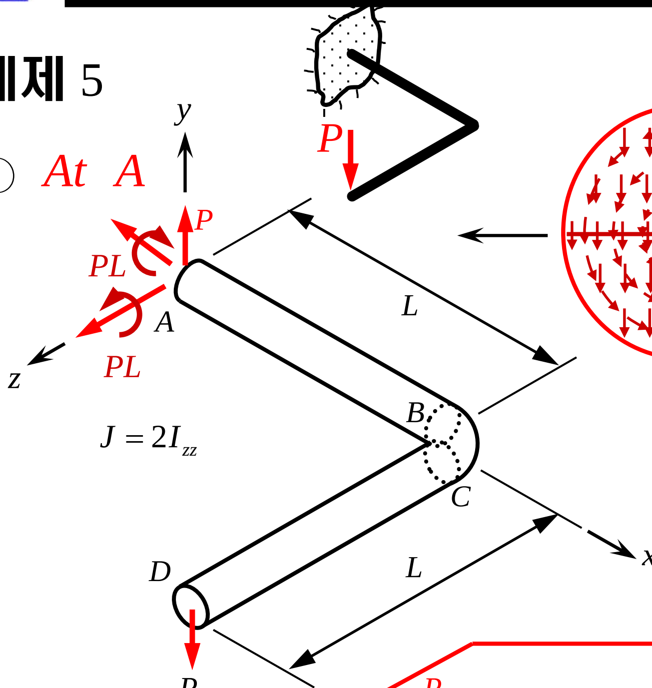


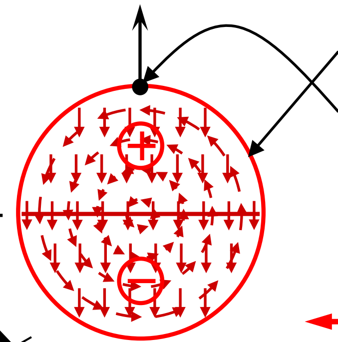
$$\bar{\sigma} = Y \rightarrow \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

굽힘모멘트와 비틀림모멘트의 복합하중

⊙ 예제 5

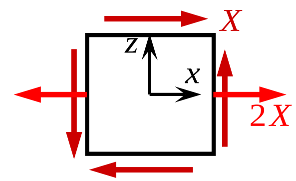
① *At A*

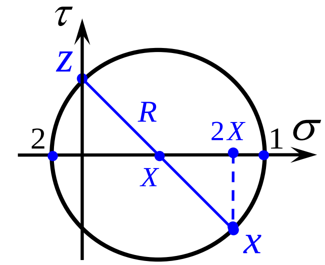




$$M_b = -PL,\ V = -P,\ M_t = PL$$

$$\sigma_x = -\frac{(-PL)\times r_0}{I_{zz}} \equiv 2X\ ,\ \ \tau_{xy} = 0,\ \ \tau_{xz} = \frac{PLr_0}{2I_{zz}} = X$$





$$\frac{PLr_0}{2I_{zz}} \equiv X$$

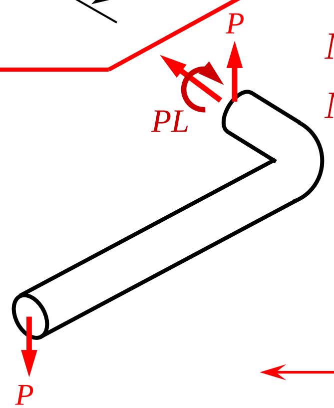
$$R = \sqrt{X^2 + X^2} = \sqrt{2}X,\ \ c = 2X/2 = X$$

$$\sigma_1 = c + R = \left(1+\sqrt{2}\right)X,\ \ \sigma_2 = \left(1-\sqrt{2}\right)X$$

$$\tau_{\max} = \frac{Y}{2} \rightarrow \sqrt{2}X = \frac{Y}{2} \rightarrow \frac{\sqrt{2}PLr_0}{2I_{zz}} = \frac{Y}{2} \rightarrow P = \frac{\sqrt{2}I_{zz}Y}{2PLr_0}$$

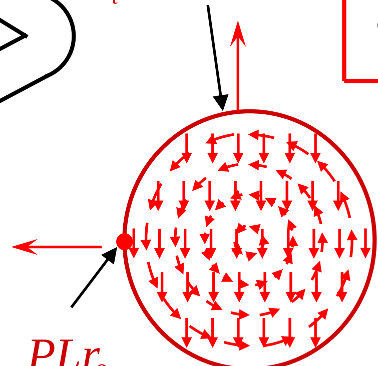
$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[8+2(1+2)X^2\right]} = \sqrt{7}X = Y \rightarrow P = \frac{2\sqrt{7}I_{zz}Y}{7PLr_0}$$

② *At B*



$$M_b = 0,\ V = -P$$

$$M_t = PL$$



$$\tau_{xy} = -\frac{PLr_0}{2I_{zz}} - \frac{PQ}{2r_0 I_{zz}}$$

$$Q = \frac{\pi r_0^{\ 2}}{2}\cdot \bar{y} = \frac{2r_0^{\ 3}}{3}$$

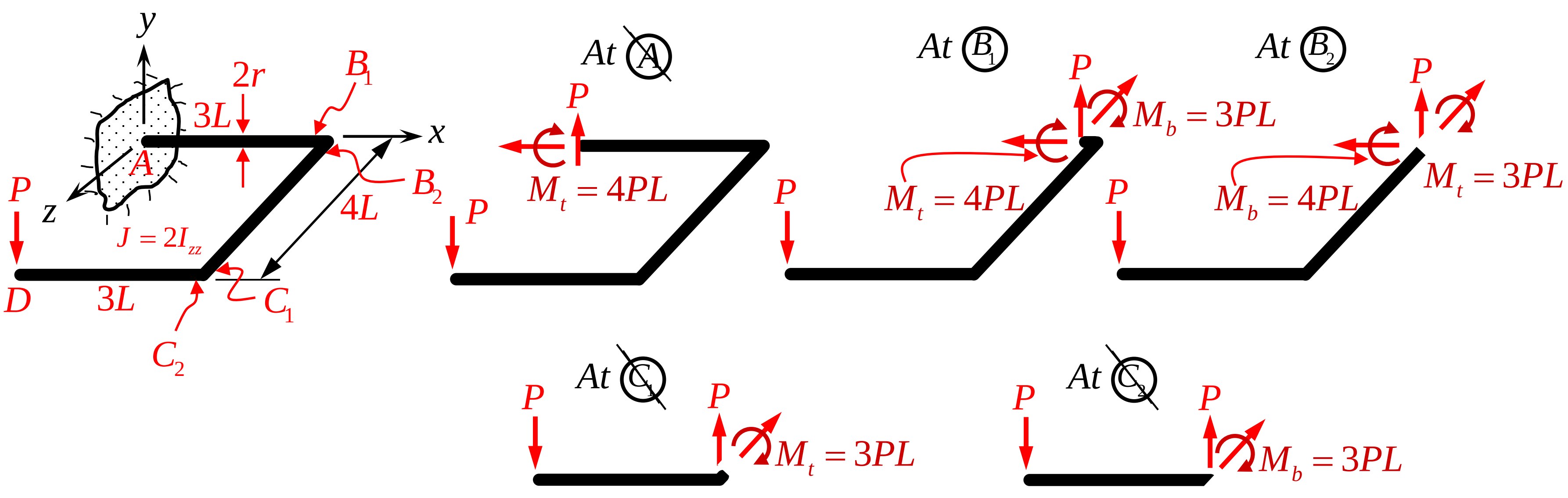
$$\bar{y} = \frac{2\int_0^{r_0} y\sqrt{r_0^{\ 2} - y^2}\,dy}{\pi r_0^{\ 2}/2} = \frac{4r_0}{3\pi}$$

$$\tau_{xy} = -\frac{PLr_0}{2I_{zz}} - \frac{Pr_0^{\ 2}}{3I_{zz}} \approx -\frac{PLr_0}{2I_{zz}}$$

점 *B* 보다 점 *A* 에 큰 응력이 작용하므로 점 *A* 에서 먼저 항복이 발생함. 설계 시,전단력 무시 가능

굽힘모멘트와 비틀림모멘트의 복합하중

⊙ 예제



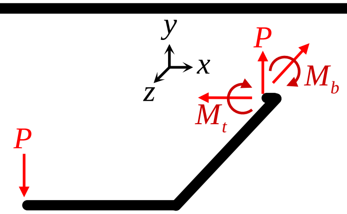
○ 소성변형 후보 점 간의 상대 비교

- 점(A) 보다 점 (B1)이 위험하다. 굽힘모멘트 $3PL$ 이 추가되었기 때문이다.
- 점(C1) 보다 점 (B2)가 위험하다. 굽힘모멘트 $4PL$ 이 추가적으로 작용하기 때문이다.
- 점(C2) 보다 점 (B1)이 위험하다. 점(B1)에 비틀림모멘트 $4PL$ 이 더 작용하기 때문이다.

∴ 점 (B1)과 점 (B2) 중의 하나에서 초기항복이 발생한다.

굽힘모멘트와 비틀림모멘트의 복합하중

⊙ 점 B_1 에서



○ 하중 및 응력:

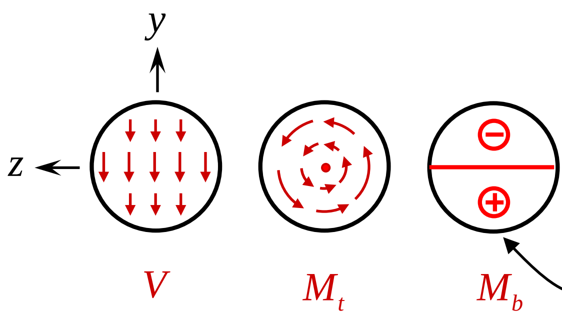
$$V=-P,\quad M_t=4PL,\quad M_b=3PL$$

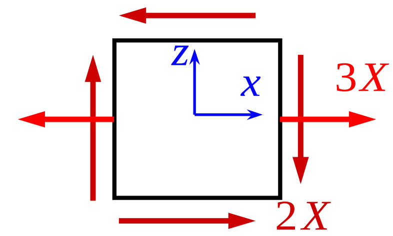
$$\left(\begin{aligned}\sigma_x &= \frac{M_b r}{I_{zz}}=\frac{3PLr}{I_{zz}}=3X\\ \tau_{xz} &= -\frac{4PLr}{2I_{zz}}=-2X\end{aligned}\right)$$

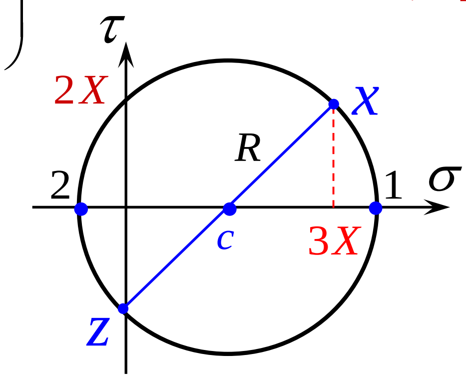
$$X\equiv\frac{PLr}{I_{zz}}$$

$J=2I_{zz}$

Weak point







$$R=\sqrt{1.5^2+2^2}X=\frac{5}{2}X,\quad c=\frac{3}{2}X$$

$$\sigma_1=c+R=4X$$

$$\sigma_2=c-R=-X$$

$$\bar{\sigma}=\sqrt{\frac{1}{2}[25+16+1]X^2}=\sqrt{21}X$$

$$\tau_{\max}=\frac{\sigma_1-\sigma_2}{2}=\frac{5}{2}X$$

○ Tresca yield criterion

$$\tau_{\max}=\frac{Y}{2}\ \rightarrow\ \frac{PLr}{I_{zz}}=\frac{Y}{5}\ \rightarrow\ P=\frac{I_{zz}Y}{5Lr}=0.2\frac{I_{zz}Y}{Lr}$$

○ von Mises yield criterion

$$P_{Mises}\geq P_{Tresca}$$

$$\bar{\sigma}=Y\ \rightarrow\ \sqrt{21}\frac{PLr}{I_{zz}}=Y\ \rightarrow\ P=0.21\frac{I_{zz}Y}{Lr}$$